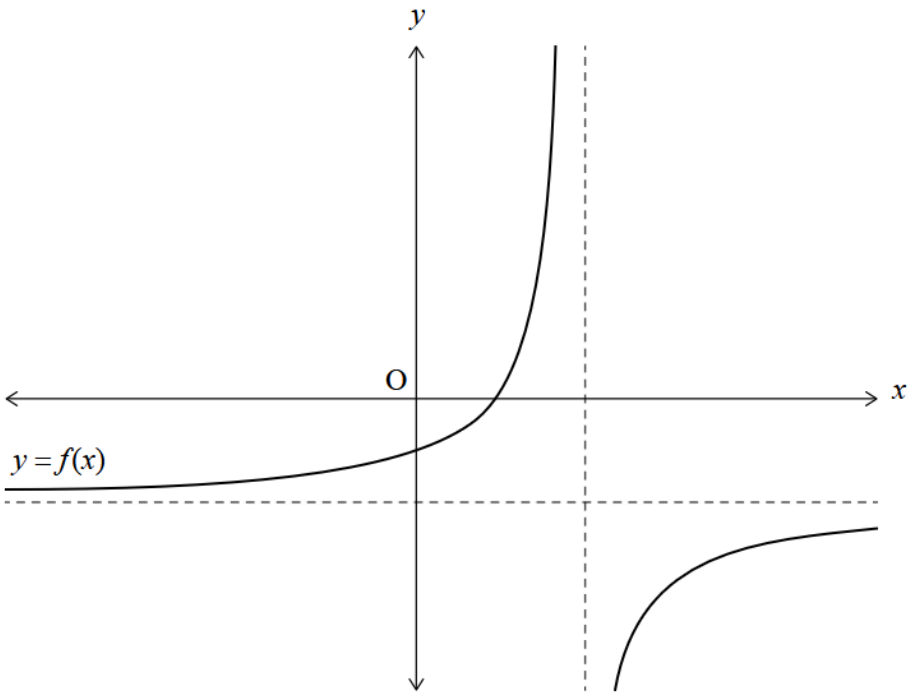
Rational Functions [194 marks]

1. [Maximum mark: 12] 26M.1.SL.TZ1.7

Consider the function $f(x) = \frac{2x-3}{3-x}$, $x \in \mathbb{R}$, $x \neq 3$.

The graph of f and its asymptotes are shown in the following diagram.



(a) Find the equation of

(a.i) the vertical asymptote; [1]

Markscheme

$x = 3$ **A1**

Note: Must see '$x =$'.

[1 mark]

(a.ii) the horizontal asymptote. [2]

Markscheme

EITHER

considers $\lim_{x \to \pm\infty} f(x)$

$\lim_{x \to \pm\infty} f(x) = -2$

OR

considers quotient of coefficients of x

$\frac{2}{-1}$

OR

attempts to divide $2x - 3$ by $3 - x$

$f(x) = -2 + \frac{3}{3-x}$

THEN

$y = -2$

Note: Must see "$y =$".

[2 marks]

(b) Find the coordinates of the points where the graph of f intersects the coordinate axes. [4]

Markscheme

attempts to solve $\frac{2x-3}{3-x} = 0$ (to find x-intercept) ***(M1)***

$2x - 3 = 0$

$\left(\frac{3}{2},\ 0\right)$ ***A1***

Note: Accept $x = \frac{3}{2}, y = 0$.

attempts to find $f(0)$ (to find y-intercept) ***(M1)***

$\frac{2(0)-3}{3-0}$

$(0, -1)$ ***A1***

Note: Accept $x = 0, y = -1$.

[4 marks]

Consider the function $g(x) = x + 4,\ \ x \in \mathbb{R}$.

(c.i) Describe the transformation that transforms the graph of $y = f(x)$ to the graph of $y = (g \circ f)(x)$. [3]

Markscheme

recognition that $(g \circ f)(x) = g(f(x))$

$(g \circ f)(x) = f(x) + 4$ ***M1***

translation 4 units upwards OR translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ ***A1A1***

Note: Award **A1** for "translation" or "shift". Do not accept "move".
Award **A1** for "4 units upwards" or "4 units in the positive vertical direction" or a vector.

[3 marks]

(c.ii) Hence, find the range of $g \circ f$. [2]

Markscheme

recognises that the horizontal asymptote is translated up 4 units ***(M1)***

$-2 + 4 = 2$

$(g \circ f)(x) \neq 2$, $\{(g \circ f)(x) < 2\} \cup \{(g \circ f)(x) > 2\}$, $y < 2$ or $y > 2$, $y < 2 \cup y > 2$ ***A1***

Note: Accept $(g \circ f)(x)$ being replaced by either $g(f(x))$ or y.

[2 marks]

2. [Maximum mark: 14] 26M.1.SL.TZ2.7

The function f is defined by $f(x) = \frac{4x-9}{2x-5}$, where $x \in \mathbb{R}$, $x \neq \frac{5}{2}$.

(a) Find the value of

(a.i) the y-intercept; [1]

Markscheme

$\frac{9}{5} (= 1.8)$ ***A1***

[1 mark]

(a.ii) the x-intercept. [1]

Markscheme

$\frac{9}{4}(=2.25)$ ***A1***

[1 mark]

(b) Find the equations of all the asymptotes of the graph of $y = f(x)$. [2]

Markscheme

$x = \frac{5}{2},\ \ y = 2$ ***A1A1***

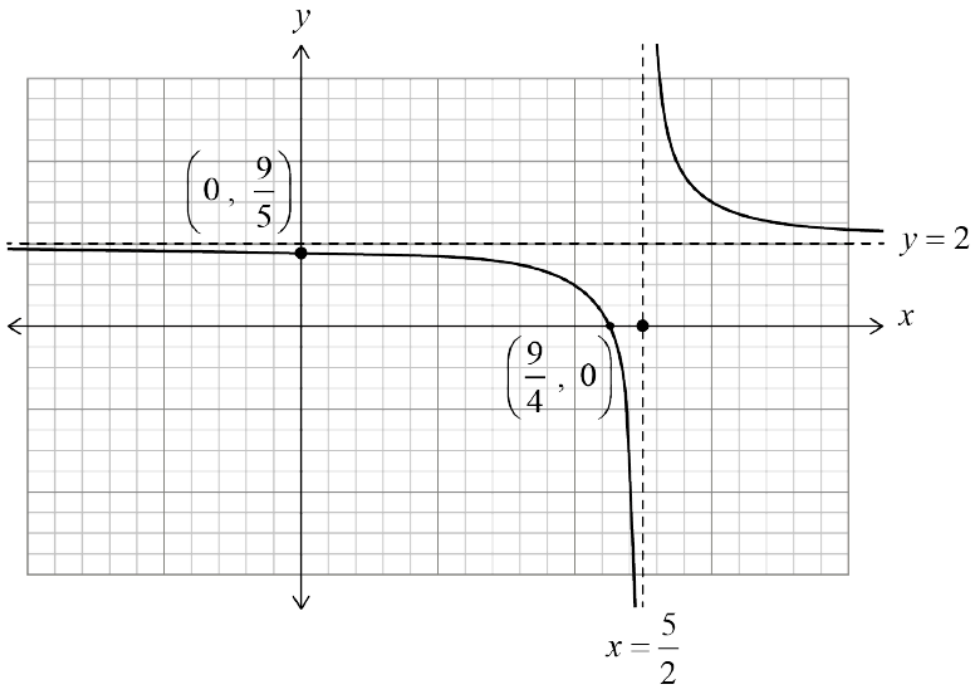
Note: If candidates use the same incorrect notation for both answers, for example, $x \neq \frac{5}{2}$ and $y \neq 2$ or $VA = \frac{5}{2}$ and $HA = 2$, then ***A0A1*** may be awarded as appropriate.

An answer like $x \neq \frac{5}{2}$ and $HA = 2$, would earn no marks.

[2 marks]

(c) Sketch the graph of $y = f(x)$, clearly showing all features found in parts (a) and (b). [2]

Markscheme



A1A1

Note: Award ***A1*** for their asymptotes and intercepts in approximately correct positions. Award ***A1*** for two branches with approximately asymptotic behaviour in the correct quadrants, Q1 and Q3, as shown above.

Intercepts and asymptotes do not have to be labelled but should be in approximately correct positions.

[2 marks]

(d) Find the inverse function, $f^{-1}(x)$, and state its domain. [4]

Markscheme

EITHER

interchanging x and y ***(M1)***

$2xy - 5x = 4y - 9$

correct working with y terms on the same side ***(A1)***

$4y - 2xy = 9 - 5x$ OR $y(4 - 2x) = 9 - 5x$

OR

correct working with x terms on the same side ***(A1)***

$4x - 2xy = 9 - 5y$

interchanging x and y or making x the subject, $x = \frac{9-5y}{4-2y}$ ***(M1)***

THEN

$f^{-1}(x) = \frac{9-5x}{4-2x}$ $\left(= \frac{5x-9}{2x-4}\right)$ or equivalent ***A1***

Note: Accept $y =$ in place of $f^{-1}(x) =$.

domain $x \neq 2$ (seen anywhere) ***A1***

Note: Award ***A1FT*** for a correct domain of their f^{-1}, **only** if their f^{-1} is a rational function.

Follow through may also be awarded from their horizontal asymptote found in part (b).

[4 marks]

(e) Hence, find the value(s) of x that satisfy $f(x) = f^{-1}(x)$. [4]

Markscheme

attempts to form a quadratic equation ***M1***

$\frac{4x-9}{2x-5} = \frac{5x-9}{2x-4}$ OR $\frac{4x-9}{2x-5} = x$ OR $\frac{5x-9}{2x-4} = x$

$2x^2 - 9x + 9 = 0$ ***A1***

attempts to solve their quadratic by factorising or formula or completing the square ***(M1)***

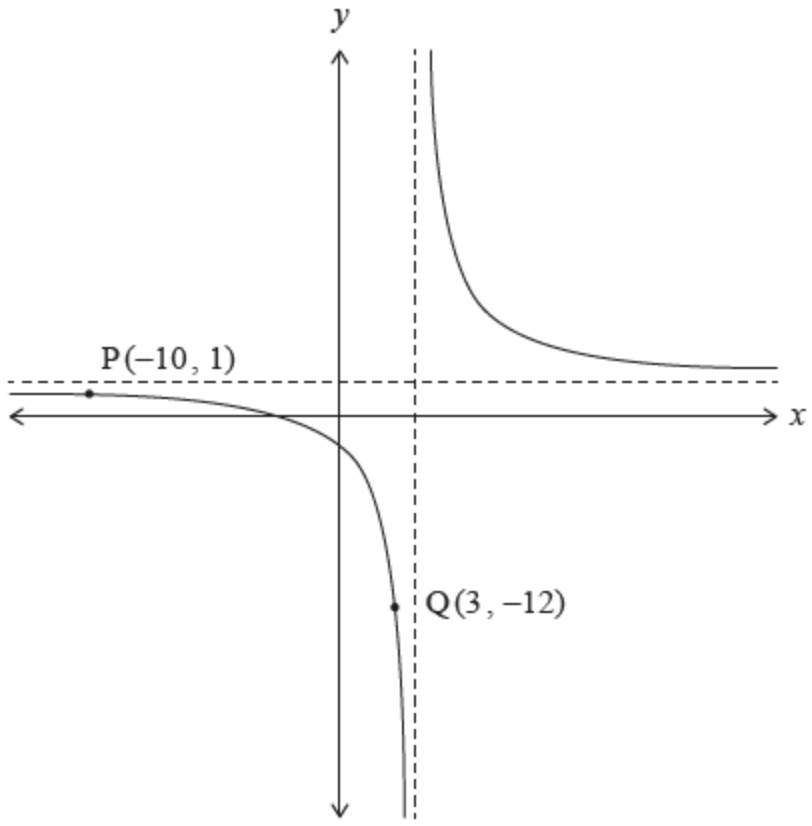
$x = \frac{3}{2},\ 3$ ***A1***

[4 marks]

3. [Maximum mark: 8] 25N.1.SL.TZ3.5

The following diagram shows the graph of $y = \frac{Ax+B}{x-4}$, where $x \in \mathbb{R}, x \neq 4$ and $A, B \in \mathbb{Z}$.

The graph passes through the points $\mathrm{P}(-10,\ 1)$ and $\mathrm{Q}(3,\ -12)$.



(a) Determine the value of A and the value of B. [3]

Markscheme

substitute $\mathrm{P}(-10,\ 1)$ or $\mathrm{Q}(3,\ -12)$ into $y = \frac{Ax+B}{x-4}$ ***(M1)***

$-14 = -10A + B$ (or equivalent)

$12 = 3A + B$ (or equivalent)

attempt to solve their equations simultaneously ***(M1)***

$A = 2, B = 6$ ***A1***

[3 marks]

(b) Describe a sequence of transformations that would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{Ax+B}{x-4}$. [5]

Markscheme

EITHER

attempt to express y in the form $P + \frac{Q}{x-4}$ ***(M1)***

OR

attempt to find one or both asymptotes using $x - 4 = 0$ or $y = \frac{a}{c}$ (dividing coefficients) ***(M1)***

THEN

recognizing vertical asymptote at $x = 4$ (may be seen anywhere, including labeled on graph or in part (a)) **OR** horizontal shift 4 units to the right ***A1***

recognizing horizontal asymptote at $y = 2$ (may be seen anywhere, including labeled on graph or in part (a)) **OR** vertical shift 2 units up
A1

$\frac{2x+6}{x-4} = 2 + \frac{14}{x-4}$ ***(A1)***

EITHER

horizontal translation 4 units to the right;
vertical stretch scale factor 14 followed by vertical translation 2 up. ***A1***

Note: Horizontal translation may be seen anywhere; y-axis transformations must be seen in the order above.

OR

vertical stretch scale factor 14; followed by translation (through/by) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$
A1

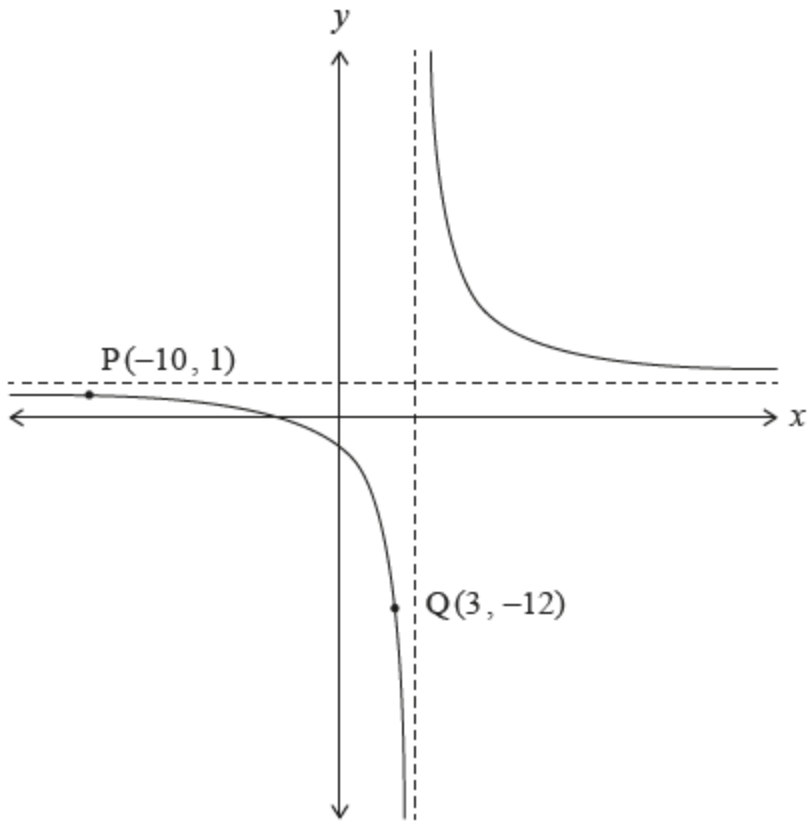
Note: In this question, the transformations may be described using terms such as *translate* or *shift*, and *dilate* or *stretch*. Do not accept "*move*" to describe a transformation.

[5 marks]

4. [Maximum mark: 8] 25N.1.AHL.TZ3.3

The following diagram shows the graph of $y = \frac{Ax+B}{x-4}$, where $x \in \mathbb{R}, x \neq 4$ and $A, B \in \mathbb{Z}$.

The graph passes through the points $\mathrm{P}(-10,\ 1)$ and $\mathrm{Q}(3,\ -12)$.



(a) Determine the value of A and the value of B. [3]

Markscheme

substitute $\mathrm{P}(-10,\ 1)$ or $\mathrm{Q}(3,\ -12)$ into $y = \frac{Ax+B}{x-4}$ ***(M1)***

$-14 = -10A + B$ (or equivalent)

$12 = 3A + B$ (or equivalent)

attempt to solve their equations simultaneously ***(M1)***

$A = 2, B = 6$ ***A1***

[3 marks]

(b) Describe a sequence of transformations that would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{Ax+B}{x-4}$. [5]

Markscheme

EITHER

attempt to express y in the form $P + \frac{Q}{x-4}$ ***(M1)***

OR

attempt to find one or both asymptotes using $x - 4 = 0$ or $y = \frac{a}{c}$ (dividing coefficients) ***(M1)***

THEN

recognizing vertical asymptote at $x = 4$ (may be seen anywhere, including labeled on graph or in part (a)) **OR** horizontal shift 4 units to the right ***A1***

recognizing horizontal asymptote at $y = 2$ (may be seen anywhere, including labeled on graph or in part (a)) **OR** vertical shift 2 units up ***A1***

$\frac{2x+6}{x-4} = 2 + \frac{14}{x-4}$ ***(A1)***

EITHER

horizontal translation 4 units to the right;
vertical stretch scale factor 14 followed by vertical translation 2 up. ***A1***

Note: Horizontal translation may be seen anywhere; y-axis transformations must be seen in the order above.

OR

vertical stretch scale factor 14; followed by translation (through/by) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

A1

Note: In this question, the transformations may be described using terms such as *translate* or *shift*, and *dilate* or *stretch*. Do not accept "*move*" to describe a transformation.

[5 marks]

5. [Maximum mark: 13] 25N.2.SL.TZ1.7

Consider the function $f(x) = \frac{3x+1}{2x-4}$ where $x \neq 2$.

(a) Write down the equation of

(a.i) the vertical asymptote of the graph of f; [1]

Markscheme

$x = 2$ ***A1***

[1 mark]

(a.ii) the horizontal asymptote of the graph of f. [1]

Markscheme

$y = \frac{3}{2}$ ***A1***

Note: Award ***A1A0*** for answers $x \neq 2$ and $y \neq \frac{3}{2}$.

[1 mark]

(b) Find $f'(x)$. [4]

Markscheme

EITHER

attempt to use the quotient rule ***(M1)***

$f'(x) = \frac{(2x-4)\times 3-(3x+1)\times 2}{(2x-4)^2}$ ***A1A1***

Note: Award ***A1*** for each term in the numerator with correct signs, provided correct denominator is seen.

OR

attempt to use the product rule ***(M1)***

$f'(x) = 3(2x-4)^{-1} + (-1)(3x+1)(2x-4)^{-2} \times 2$ ***A1A1***

Note: Award ***A1*** for each term.

THEN

$f'(x) = \frac{-14}{(2x-4)^2}$ ***A1***

[4 marks]

(c) Hence, explain briefly why f is never increasing. [2]

Markscheme

recognising $f'(x) < 0$ (when f is decreasing) **OR** $(2x-4)^2 > 0$ **OR** the gradient (function) is negative (may be seen on a labelled graph of f') ***A1***

for all values of x (with $x \neq 2$) **OR** always (or equivalent) ***A1***

f is never increasing ***AG***

Note: Do not accept a sketch and argument based on the graph of f.

[2 marks]

The point $\mathrm{P}(x,\ f(x))$ is on the graph of f such that $x < 2$.

Let d be the distance between the points P and $\mathrm{Q}(3,\ 5)$.

(d) Find an expression for d in terms of x. [2]

Markscheme

attempt to use the distance formula ***(M1)***

$d = \sqrt{(x-3)^2 + \left(\frac{3x+1}{2x-4} - 5\right)^2}$ ***A1***

Note: Accept $d^2 = (x-3)^2 + (f(x)-5)^2$.

[2 marks]

(e) Hence, find the coordinates of P such that d is a minimum. [3]

Markscheme

attempt to find value of x that gives minimum value of d ***(M1)***

graph of d, $d' = 0$,

$(x =) -0.305218\ldots, (f(-0.305218\ldots) =) -0.0182941\ldots$

P has coordinates $(-0.305, \ -0.0183)$ ***A1A1***

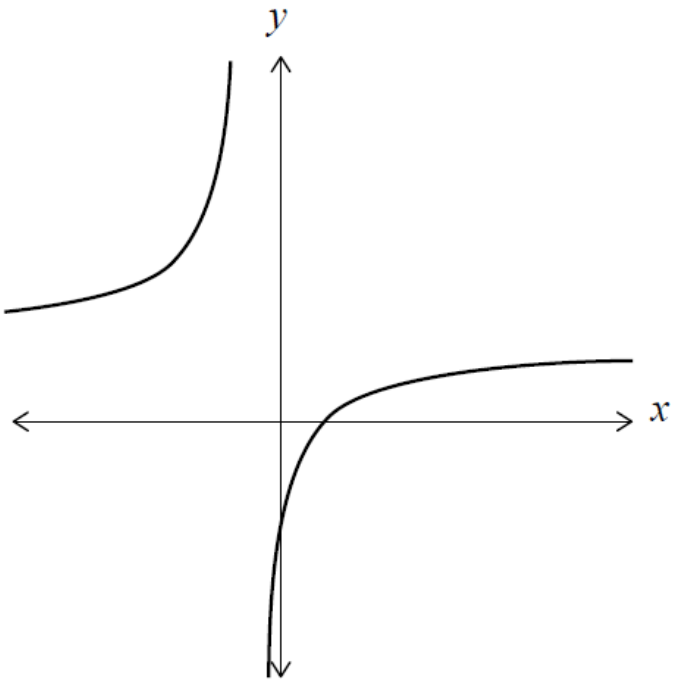
Note: Award ***A1*** for each value.

[3 marks]

6. [Maximum mark: 5] 25M.1.SL.TZ2.3

The function f is defined by $f(x) = \frac{3x-2}{2x+1}$ for $x \in \mathbb{R}, x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.



(a) Write down the value of $f(0)$. [1]

Markscheme

-2 (accept $(0, -2)$) ***A1***

[1 mark]

(b) Write down the equation of the horizontal asymptote. [1]

Markscheme

$y = \frac{3}{2}$ (must be an equation) ***A1***

Note: Do not accept $\neq$ sign.

[1 mark]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

Markscheme

attempt to reflect the graph of f in the x-axis **OR** one correct value seen ***(M1)***

both correct values $-\frac{3}{2}$ and 2 (seen anywhere) ***A1***

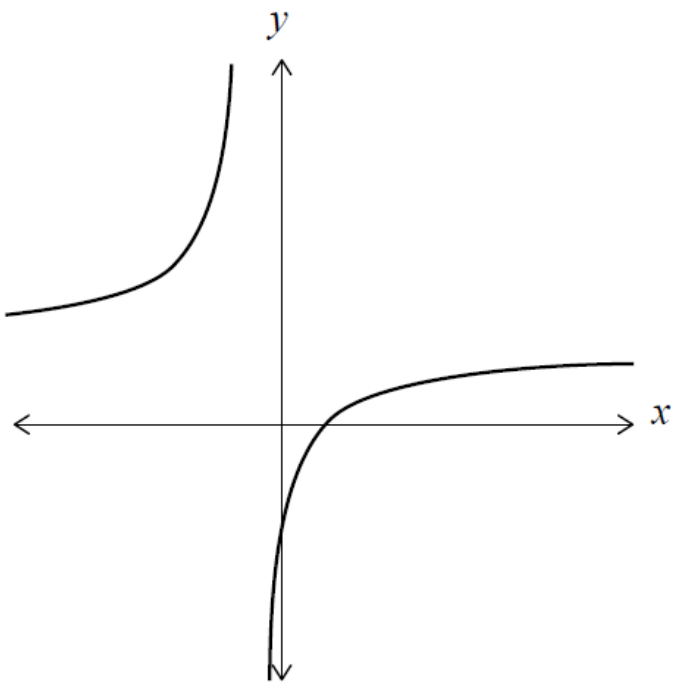
$-\frac{3}{2} < y \leq 2$ ***A1***

[3 marks]

7. [Maximum mark: 5] 25M.1.AHL.TZ2.1

The function f is defined by $f(x) = \frac{3x-2}{2x+1}$ for $x \in \mathbb{R}, x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.



(a) Write down the value of $f(0)$. [1]

Markscheme

-2 (accept $(0, -2)$) ***A1***

[1 mark]

(b) Write down the equation of the horizontal asymptote. [1]

Markscheme

$y = \frac{3}{2}$ (must be an equation) ***A1***

Note: Do not accept $\neq$ sign.

[1 mark]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

Markscheme

attempt to reflect the graph of f in the x-axis **OR** one correct value seen ***(M1)***

both correct values $-\frac{3}{2}$ and 2 (seen anywhere) ***A1***

$-\frac{3}{2} < y \leq 2$ ***A1***

[3 marks]

8. [Maximum mark: 15] 25M.2.SL.TZ1.9

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R},\ x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

Markscheme

METHOD 1

attempt to rearrange and swap x and y (at any stage) ***(M1)***

$x = \frac{2-2y}{y+2} \Rightarrow xy + 2x = -2y + 2$

$xy + 2y = 2 - 2x$ **OR** $y(x+2) = 2 - 2x$ ***A1***

$f^{-1}(x) = \frac{2-2x}{x+2}$ **OR** $y = \frac{2-2x}{x+2}$ ***A1***

So $f^{-1}(x) = f(x)$ ***AG***

METHOD 2

attempt to find $f(f(x))$ ***(M1)***

$f(f(x)) = \frac{2-2\left(\frac{2-2x}{x+2}\right)}{\left(\frac{2-2x}{x+2}\right)+2}$

$= \frac{2(x+2)-2(2-2x)}{(2-2x)+2(x+2)} \left(= \frac{6x}{6}\right)$ ***A1***

$= x$ ***A1***

So $f^{-1}(x) = f(x)$ ***AG***

[3 marks]

The point $\mathrm{P}\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0,\ 0)$.

(b.i) Find the value of k. [3]

Markscheme

EITHER

$\mathrm{OP} = \sqrt{k^2 + \left(\frac{2-2k}{k+2}\right)^2}$ ***(A1)***

attempt to minimise distance ***(M1)***

OR

recognition that P lies on $y = x$ due to symmetry ***(M1)***

$k = -4.44948\ldots$ or $k = 0.449489\ldots$ ***(A1)***

THEN

minimum when $k = 0.449489\ldots$

$k = 0.449$ ***A1***

Note: If no working seen, award ***(M0)(A0)A0*** for 0.45.

[3 marks]

(b.ii) Hence, write down the coordinates of P. [1]

Markscheme

coordinates of P $(0.449,\ 0.449)$ ***A1***

Note: Award ***A1FT*** for $f(k)$, using their k.

[1 mark]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R},\ x \neq -\frac{d}{c}$, and $c,\ d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d, write down the equation of

(c.i) the vertical asymptote; [1]

Markscheme

$x = -\frac{d}{c}$ ***A1***

Note: Answers in (i) and (ii) must be correct equations.

[1 mark]

(c.ii) the horizontal asymptote. [1]

Markscheme

$y = -\frac{3}{c}$ ***A1***

Note: Answers in (i) and (ii) must be correct equations.

[1 mark]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d. [2]

Markscheme

$g^{-1}(x) = \frac{2-dx}{cx+3}$ **OR** $-\frac{d}{c} = -\frac{3}{c}$ ***(A1)***

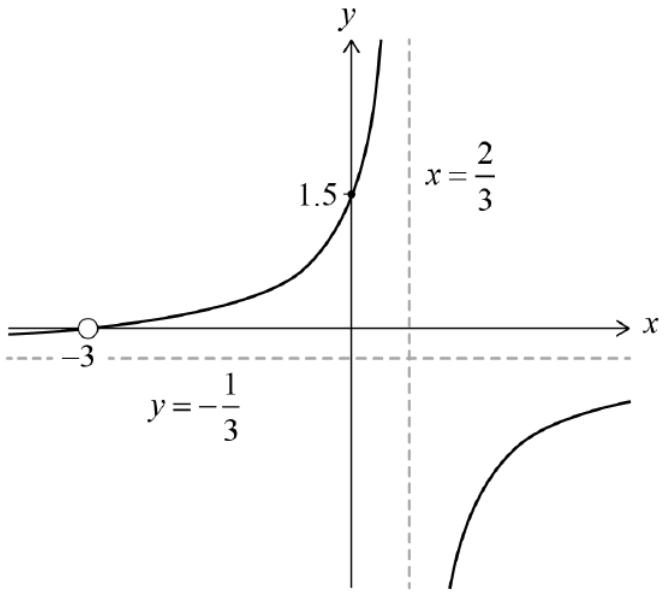
$d = 3$ ***A1***

[2 marks]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]

Markscheme



Only if shape is approximately correct for this rational function (i.e. two branches in opposite corners and displaying appropriate asymptotic behaviour), award the following marks independently.

asymptotes $x = \frac{2}{3}$ and $y = -\frac{1}{3}$ drawn with correct labels ***A1A1***

labelled point discontinuity (or x-intercept) at $(-3,\ 0)$, labelled y-intercept at $\left(0,\ \frac{3}{2}\right)$ ***A1A1***

Note: Award ***A1FTA1FT***, as appropriate, for their value of d (i.e. x-intercept $= -d$, y-intercept $\frac{d}{2}$).

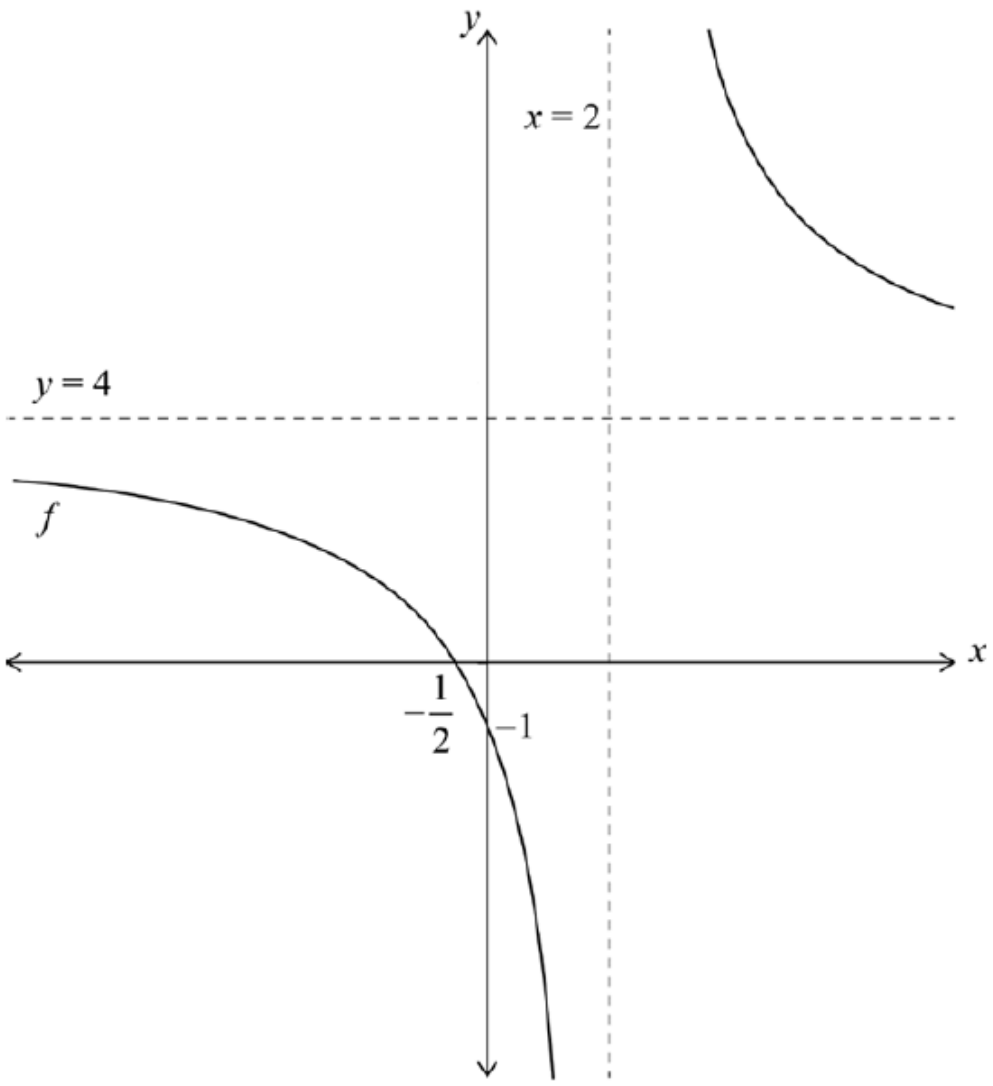
[4 marks]

9. [Maximum mark: 14] 24M.1.SL.TZ1.8

Consider the function $f(x) = \frac{4x+2}{x-2},\ x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

Markscheme



vertical asymptote $x = 2$ sketched and labelled with correct equation

A1

horizontal asymptote $y = 4$ sketched and labelled with correct equation

A1

For an approximate rational function shape:

labelled intercepts $-\frac{1}{2}$ on x-axis, -1 on y-axis ***A1A1***

two branches in correct opposite quadrants with correct asymptotic behaviour ***A1***

Note: These marks may be awarded independently.

[5 marks]

(b) Write down the range of f. [1]

Markscheme

$y \neq 4$ (or equivalent) ***A1***

[1 mark]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

Markscheme

$2 + \frac{5}{2}$ OR $\left(-\frac{1}{2}\right) + 2 \times \frac{5}{2}$ OR $\frac{-\frac{1}{2}+p}{2} = 2$ OR $-4 = -p + \frac{1}{2}$ ***A1***

$p = \frac{9}{2}$ ***AG***

[1 mark]

(d) Find the value of b and the value of c. [3]

Markscheme

METHOD 1

attempt to substitute both roots to form a quadratic ***(M1)***

EITHER

$\left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$ OR $x^2-\left(-\frac{1}{2}+\frac{9}{2}\right)x+\left(-\frac{1}{2}\times\frac{9}{2}\right)$

$=x^2-4x-\frac{9}{4}$ ***A1A1***

$\left(b=-4, c=-\frac{9}{4}\right)$

Note: Award ***A1*** for each correct value. They may be embedded or stated explicitly.

OR

$(2x+1)(2x-9)=4\left(x^2-4x-\frac{9}{4}\right)$

$b=-4, c=-\frac{9}{4}$ ***A1A1***

Note: Award ***A1*** for each correct value. They must be stated explicitly.

METHOD 2

$-\frac{b}{2}=2$ OR $4+b=0\Rightarrow b=-4$ ***A1***

attempt to form a valid equation to find c using their b ***(M1)***

$\left(-\frac{1}{2}\right)^2 + -4\left(-\frac{1}{2}\right) + c = 0$ OR $\left(\frac{9}{2}\right)^2 + -4\left(\frac{9}{2}\right) + c = 0$

$c = -\frac{9}{4}$ ***A1***

METHOD 3

attempt to form two valid equations in b and c ***(M1)***

$\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + c = 0,\ \left(\frac{9}{2}\right)^2 + b\left(\frac{9}{2}\right) + c = 0$

$b = -4, c = -\frac{9}{4}$ ***A1A1***

METHOD 4

attempt to write $g(x)$ in the form $(x-h)^2 + k$ and substitute for x, h and $g(x)$ ***(M1)***

$\left(-\frac{1}{2} - 2\right)^2 + k = 0 \Rightarrow k = -\frac{25}{4}$

$(x-2)^2 - \frac{25}{4}$

$= x^2 - 4x - \frac{9}{4}$ ***A1A1***

$\left(b = -4, c = -\frac{9}{4}\right)$

Note: Award ***A1*** for each correct value. They may be embedded or stated explicitly.

[3 marks]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

Markscheme

attempt to substitute $x = 2$ into their $g(x)$ OR

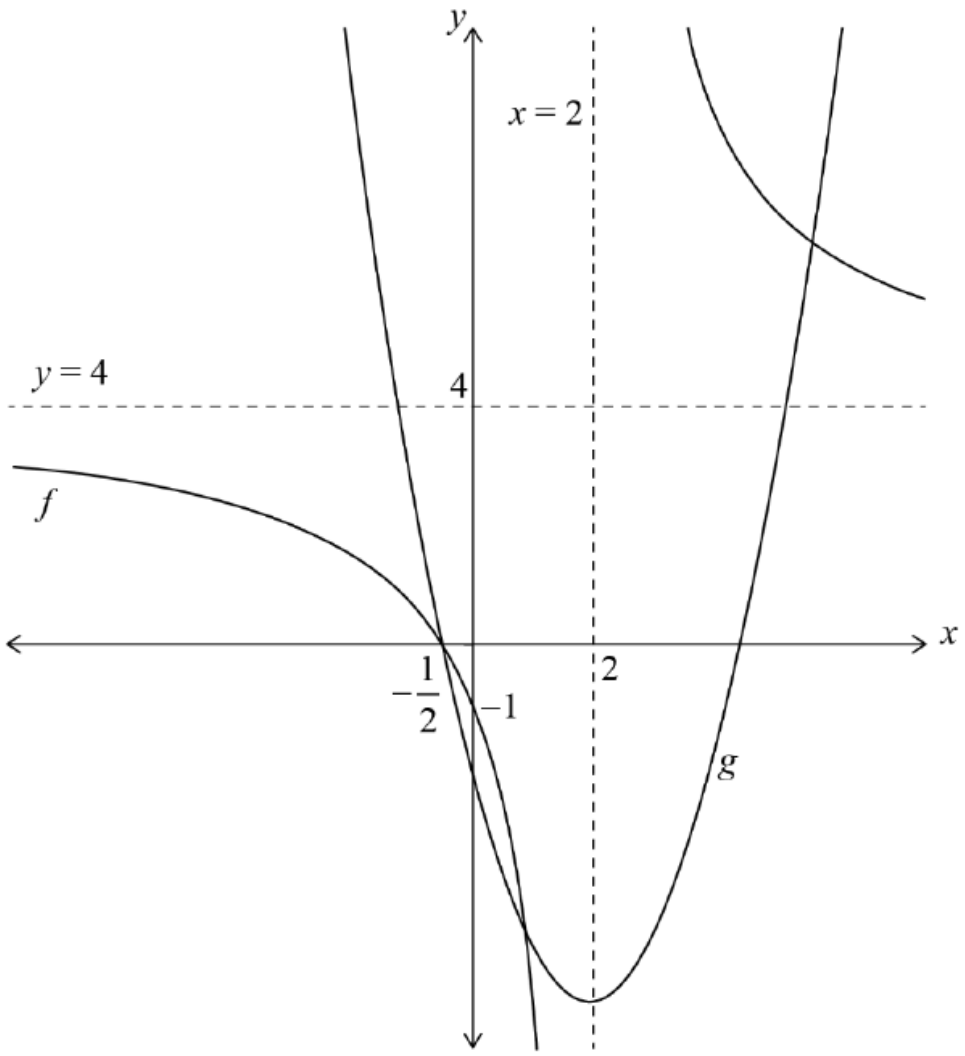
complete the square on their $g(x)$ (may be seen in part (d)) ***(M1)***

$y = -\frac{25}{4}$ ***A1***

[2 marks]

(f) Find the number of solutions of the equation $f(x) = g(x)$. [2]

Markscheme



both graphs sketched on same axes and identifying points of intersection
(M1)

3 solutions ***A1***

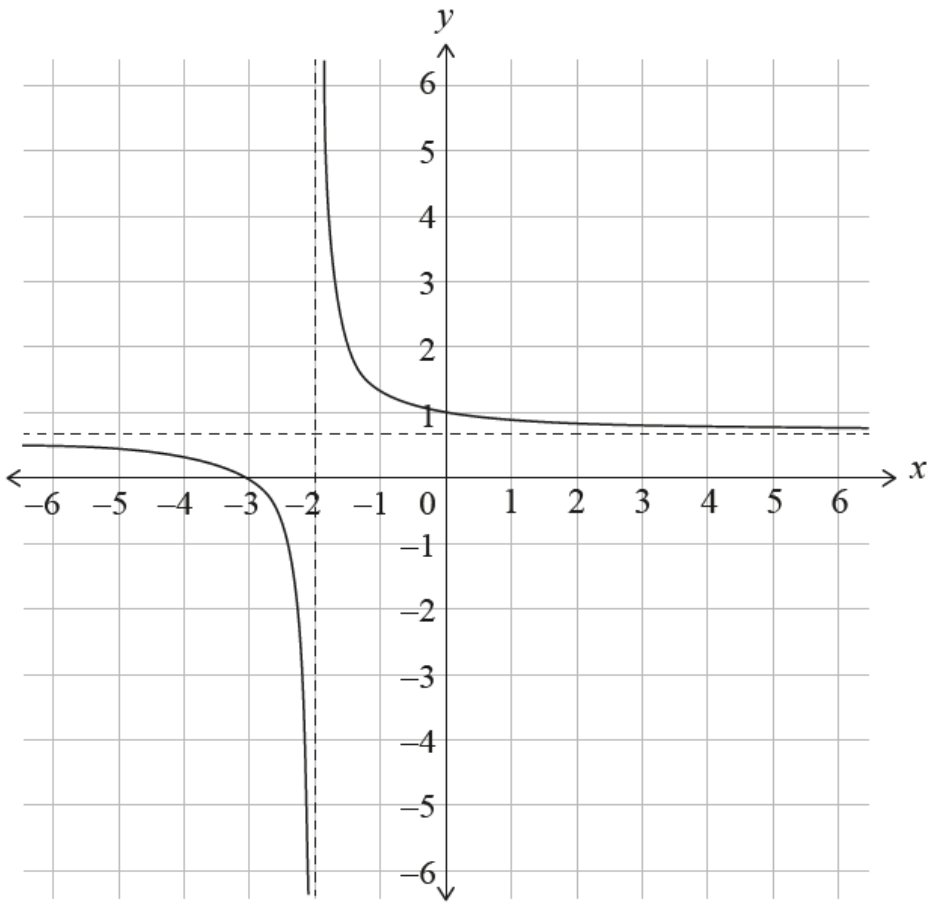
Note: Exception to ***FT***: If the candidate's graph in part (a) is incorrect, the ***M1*** may be awarded for a sketch of their graph from part (a) and $g(x)$. Do not award the final ***A1*** in this case.

[2 marks]

10. [Maximum mark: 8] 24M.1.SL.TZ2.6

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R},\ x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Markscheme

$y = \frac{2}{3}$ (must be written as equation with $y =$) ***A1***

[1 mark]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R},\ m \neq 0$.

(b.i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$. [1]

Markscheme

2 ***A1***

[1 mark]

(b.ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x. [4]

Markscheme

EITHER

$\frac{2(x+3)}{3(x+2)} = mx + 1$

attempt to expand to obtain a quadratic equation ***(M1)***

$2x + 6 = 3mx^2 + 6mx + 3x + 6$

$3mx^2 + (6m + 1)x = 0$ OR $3mx^2 + 6mx + x = 0$ ***A1***

recognition that discriminant $\Delta = 0$ for one solution ***(M1)***

$(6m + 1)^2 = 0$

OR

$\frac{2(x+3)}{3(x+2)} = mx + 1$

attempt to expand to obtain a quadratic equation ***(M1)***

$2x + 6 = 3mx^2 + 6mx + 3x + 6$

$3mx^2 + (6m + 1)x = 0$ OR $3mx^2 + 6mx + x = 0$ ***A1***

attempt to solve their quadratic for x and equating their solutions ***(M1)***

$x(3mx + 6m + 1) = 0$

$x = 0$ OR $x = -\frac{6m+1}{3m} (= 0)$

$-\frac{6m+1}{3m} = 0$

OR

attempt to find $f\prime(x)$ using the quotient rule ***(M1)***

$f\prime(x) = \frac{2}{3}\left(\frac{(x+2)-(x+3)}{(x+2)^2}\right) = \left(\frac{-2}{3(x+2)^2}\right)$ OR $\frac{2(3x+6)-3(2x+6)}{(3x+6)^2}$ or equivalent ***A1***

recognition that m is the derivative of $f(x)$ at $x = 0$ ***(M1)***

THEN

$\Rightarrow m = -\frac{1}{6}$ ***A1***

[4 marks]

(b.iii) Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$. [2]

Markscheme

Note: In this part, FT may be awarded only for values of m between -1 and 0.

$-\frac{1}{6} < m < 0$ ***A2***

Note: Award ***A1*** for only $m > -\frac{1}{6}$. Award ***A1*** for only $m < 0$.

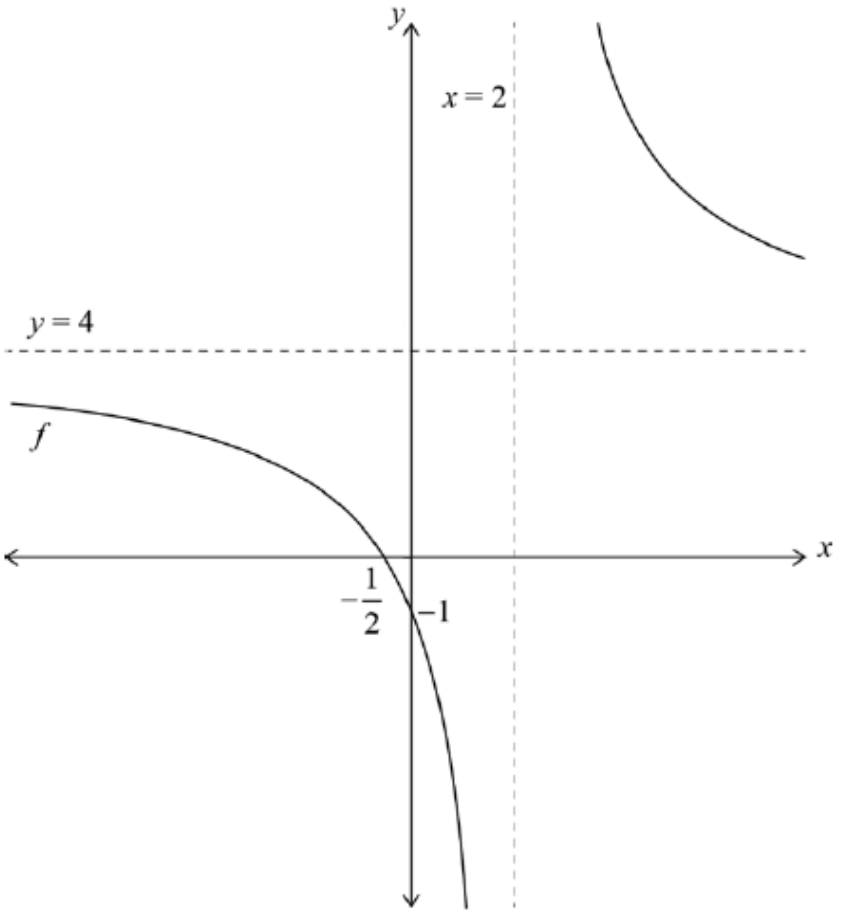
[2 marks]

11. [Maximum mark: 16] 24M.1.AHL.TZ1.10

Consider the function $f(x) = \frac{4x+2}{x-2},\ x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

Markscheme



vertical asymptote $x = 2$ sketched and labelled with correct equation

A1

horizontal asymptote $y = 4$ sketched and labelled with correct equation ***A1***

For an approximate rational function shape:

labelled intercepts $-\frac{1}{2}$ on x-axis, -1 on y-axis ***A1A1***

two branches in correct opposite quadrants with correct asymptotic behaviour ***A1***

Note: These marks may be awarded independently.

[5 marks]

(b) Write down the range of f. [1]

Markscheme

$y \neq 4$ (or equivalent) ***A1***

[1 mark]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

Markscheme

$2 + \frac{5}{2}$ OR $\left(-\frac{1}{2}\right) + 2 \times \frac{5}{2}$ OR $\frac{-\frac{1}{2}+p}{2} = 2$ OR $-4 = -p + \frac{1}{2}$ ***A1***

$p = \frac{9}{2}$ ***AG***

[1 mark]

(d) Find the value of b and the value of c. [3]

Markscheme

METHOD 1

attempt to substitute both roots to form a quadratic ***(M1)***

EITHER

$\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right)$ OR $x^2 - \left(-\frac{1}{2} + \frac{9}{2}\right)x + \left(-\frac{1}{2} \times \frac{9}{2}\right)$

$= x^2 - 4x - \frac{9}{4}$ ***A1A1***

$\left(b = -4, c = -\frac{9}{4}\right)$

Note: Award ***A1*** for each correct value. They may be embedded or stated explicitly.

OR

$(2x + 1)(2x - 9) = 4\left(x^2 - 4x - \frac{9}{4}\right)$

$b = -4, c = -\frac{9}{4}$ ***A1A1***

Note: Award ***A1*** for each correct value. They must be stated explicitly.

METHOD 2

$-\frac{b}{2} = 2$ OR $4 + b = 0 \Rightarrow b = -4$ ***A1***

attempt to form a valid equation to find c using their b ***(M1)***

$\left(-\frac{1}{2}\right)^2 + -4\left(-\frac{1}{2}\right) + c = 0$ OR $\left(\frac{9}{2}\right)^2 + -4\left(\frac{9}{2}\right) + c = 0$

$c = -\frac{9}{4}$ ***A1***

METHOD 3

attempt to form two valid equations in b and c ***(M1)***

$\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + c = 0,\ \left(\frac{9}{2}\right)^2 + b\left(\frac{9}{2}\right) + c = 0$

$b = -4, c = -\frac{9}{4}$ ***A1A1***

METHOD 4

attempt to write $g(x)$ in the form $(x - h)^2 + k$ and substitute for x, h and $g(x)$ ***(M1)***

$\left(-\frac{1}{2} - 2\right)^2 + k = 0 \Rightarrow k = -\frac{25}{4}$

$(x - 2)^2 - \frac{25}{4}$

$= x^2 - 4x - \frac{9}{4}$ ***A1A1***

$\left(b = -4, c = -\frac{9}{4}\right)$

Note: Award ***A1*** for each correct value. They may be embedded or stated explicitly.

[3 marks]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

Markscheme

attempt to substitute $x = 2$ into their $g(x)$ OR

complete the square on their $g(x)$ (may be seen in part (d)) ***(M1)***

$y = -\frac{25}{4}$ ***A1***

[2 marks]

(f) Find the product of the solutions of the equation $f(x) = g(x)$. [4]

Markscheme

$\frac{4x+2}{x-2} = \left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right)$ OR $\frac{4x+2}{x-2} = x^2 - 4x - \frac{9}{4}$

attempt to form a cubic equation ***(M1)***

EITHER

$4x + 2 = (x - 2)\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right)$ OR
$4x + 2 = \left(x^2 - 4x - \frac{9}{4}\right)(x - 2)$ OR

$(x - 2)\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right) - 4x - 2$ OR
$(x - 2)\left(x^2 - 4x - \frac{9}{4}\right) - 4x - 2$

$x^3 + \ldots + \frac{5}{2} (= 0)$ OR $4x^3 + \ldots + 10 (= 0)$ ***(A1)(A1)***

Note: Award ***(A1)*** for each of the terms x^3 and $\frac{5}{2}$ or $4x^3$ and 10. Ignore extra terms.

product of roots $= \left(\frac{(-1)^3 \times \frac{5}{2}}{1}\right)$ OR $\left(\frac{(-1)^3 \times 10}{4}\right)$

$= -\frac{5}{2}$ ***A1***

OR

$4\left(x + \frac{1}{2}\right) = (x-2)\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right)$

$x = -\frac{1}{2}$ ***(A1)***

or $4 = x^2 + \ldots + 9 \Rightarrow x^2 + \ldots + 5 = 0$

product of roots of quadratic is 5 ***(A1)***

product is therefore $-\frac{1}{2} \times 5$

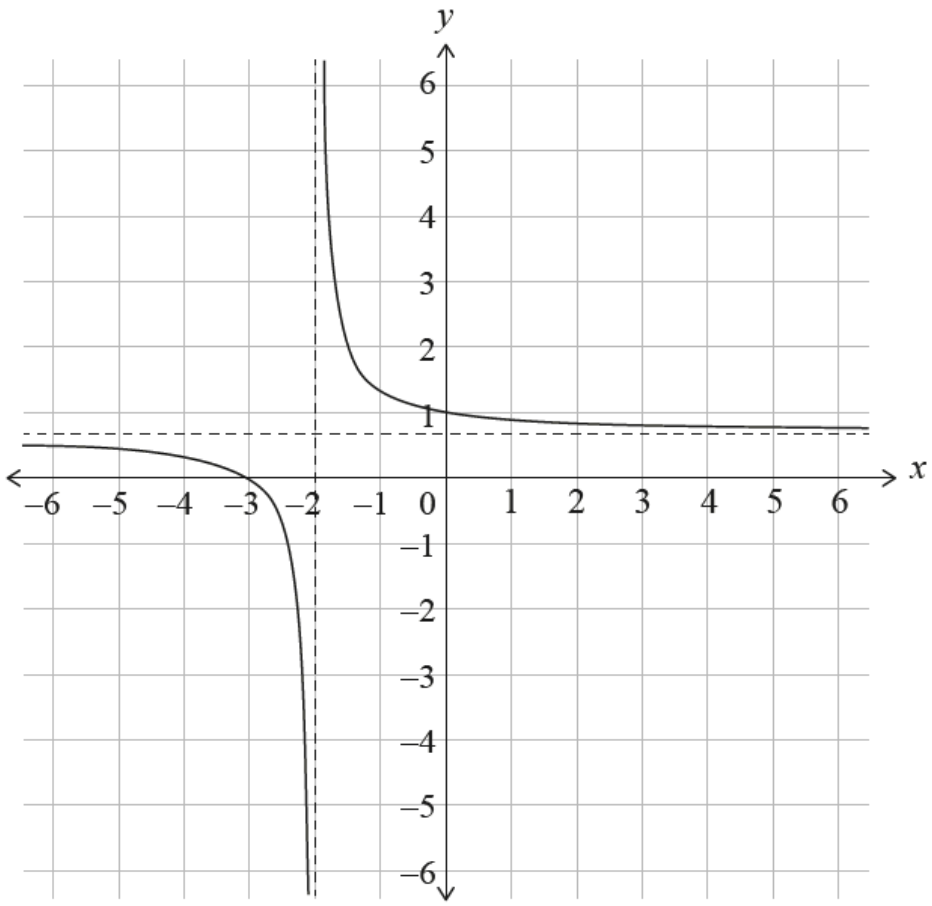
$= -\frac{5}{2}$ ***A1***

[4 marks]

12. [Maximum mark: 8] 24M.1.AHL.TZ2.5

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R},\ x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Markscheme

$y = \frac{2}{3}$ (must be written as equation with $y =$) ***A1***

[1 mark]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R},\ m \neq 0$.

(b.i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$. [1]

Markscheme

2 ***A1***

[1 mark]

(b.ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x. [4]

Markscheme

EITHER

$\frac{2(x+3)}{3(x+2)} = mx + 1$

attempt to expand to obtain a quadratic equation ***(M1)***

$2x + 6 = 3mx^2 + 6mx + 3x + 6$

$3mx^2 + (6m + 1)x = 0$ OR $3mx^2 + 6mx + x = 0$ ***A1***

recognition that discriminant $\Delta = 0$ for one solution ***(M1)***

$(6m + 1)^2 = 0$

OR

$\frac{2(x+3)}{3(x+2)} = mx + 1$

attempt to expand to obtain a quadratic equation ***(M1)***

$2x + 6 = 3mx^2 + 6mx + 3x + 6$

$3mx^2 + (6m + 1)x = 0$ OR $3mx^2 + 6mx + x = 0$ ***A1***

attempt to solve their quadratic for x and equating their solutions ***(M1)***

$x(3mx + 6m + 1) = 0$

$x = 0$ OR $x = -\frac{6m+1}{3m} (= 0)$

$-\frac{6m+1}{3m} = 0$

OR

attempt to find $f\prime(x)$ using the quotient rule ***(M1)***

$f\prime(x) = \frac{2}{3}\left(\frac{(x+2)-(x+3)}{(x+2)^2}\right) = \left(\frac{-2}{3(x+2)^2}\right)$ OR $\frac{2(3x+6)-3(2x+6)}{(3x+6)^2}$ or equivalent ***A1***

recognition that m is the derivative of $f(x)$ at $x = 0$ ***(M1)***

THEN

$\Rightarrow m = -\frac{1}{6}$ ***A1***

[4 marks]

(b.iii) Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$. [2]

Markscheme

Note: In this part, FT may be awarded only for values of m between -1 and 0.

$-\frac{1}{6} < m < 0$ ***A2***

Note: Award ***A1*** for only $m > -\frac{1}{6}$. Award ***A1*** for only $m < 0$.

[2 marks]

13. [Maximum mark: 7] 23M.1.SL.TZ1.2

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

Markscheme

recognizing $f(x) = 0$ ***(M1)***

$x = -1$ ***A1***

[2 marks]

(b) For the graph of $y = f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

Markscheme

$x = 2$ (must be an equation with x) ***A1***

[1 mark]

(b.ii) the horizontal asymptote. [1]

Markscheme

$y = \frac{7}{2}$ (must be an equation with y) ***A1***

[1 mark]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

Markscheme

EITHER

interchanging x and y ***(M1)***

$2xy - 4x = 7y + 7$

correct working with y terms on the same side: $2xy - 7y = 4x + 7$ ***(A1)***

OR

$2yx - 4y = 7x + 7$

correct working with x terms on the same side: $2yx - 7x = 4y + 7$ ***(A1)***

interchanging x and y OR making x the subject $x = \frac{4y+7}{2y-7}$ ***(M1)***

THEN

$f^{-1}(x) = \frac{4x+7}{2x-7}$ (or equivalent) $\left(x \neq \frac{7}{2}\right)$ ***A1***

[3 marks]

14. [Maximum mark: 5] 23M.1.SL.TZ2.3

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(a.i) the vertical asymptote; [1]

Markscheme

$x = 2$ ***A1***

[1 mark]

(a.ii) the horizontal asymptote. [1]

Markscheme

$y = 1$ ***A1***

[1 mark]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(b.i) the y-axis; [1]

Markscheme

$\left(0, \frac{3}{2}\right)$ ***A1***

[1 mark]

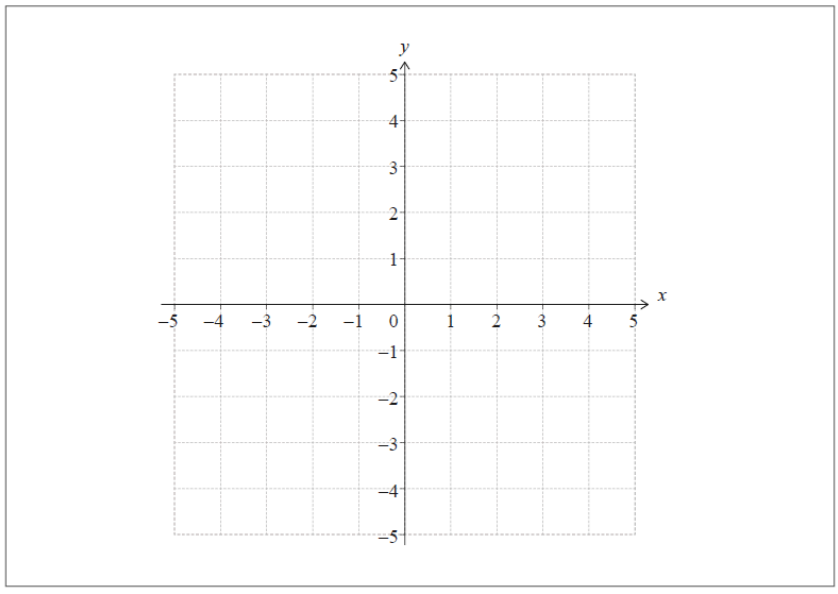
(b.ii) the x-axis. [1]

Markscheme

$(3, 0)$ ***A1***

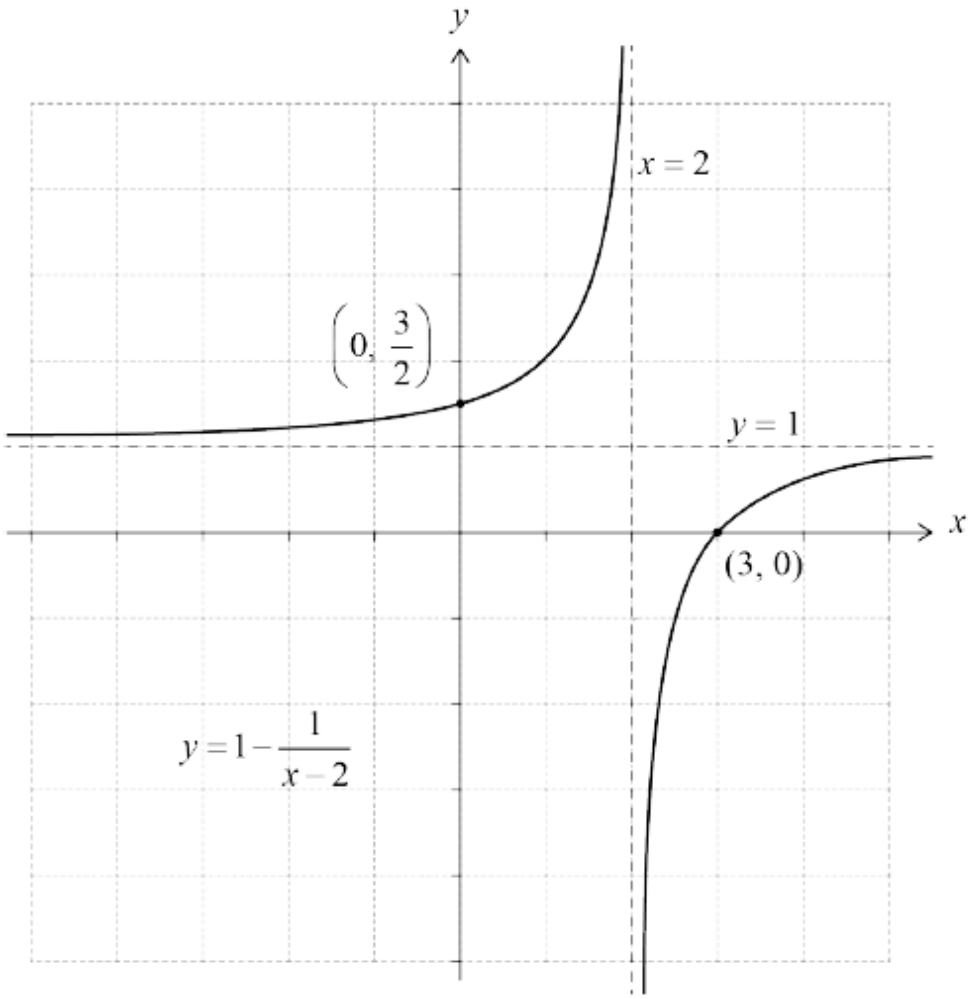
[1 mark]

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).



[1]

Markscheme



two correct branches with correct asymptotic behaviour and intercepts clearly shown **A1**

[1 mark]

15. [Maximum mark: 7] 23M.1.AHL.TZ1.1

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

Markscheme

recognizing $f(x) = 0$ ***(M1)***

$x = -1$ ***A1***

[2 marks]

(b) For the graph of $y = f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

Markscheme

$x = 2$ (must be an equation with x) ***A1***

[1 mark]

(b.ii) the horizontal asymptote. [1]

Markscheme

$y = \frac{7}{2}$ (must be an equation with y) ***A1***

[1 mark]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

Markscheme

EITHER

interchanging x and y ***(M1)***

$2xy - 4x = 7y + 7$

correct working with y terms on the same side: $2xy - 7y = 4x + 7$
(A1)

OR

$2yx - 4y = 7x + 7$

correct working with x terms on the same side: $2yx - 7x = 4y + 7$
(A1)

interchanging x and y OR making x the subject $x = \frac{4y+7}{2y-7}$ ***(M1)***

THEN

$f^{-1}(x) = \frac{4x+7}{2x-7}$ (or equivalent) $\left(x \neq \frac{7}{2}\right)$ ***A1***

[3 marks]

16. [Maximum mark: 5] 23M.1.AHL.TZ2.2

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}, x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(a.i) the vertical asymptote; [1]

Markscheme

$x = 2$ **A1**

[1 mark]

(a.ii) the horizontal asymptote. [1]

Markscheme

$y = 1$ **A1**

[1 mark]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(b.i) the y-axis; [1]

Markscheme

$\left(0, \frac{3}{2}\right)$ **A1**

[1 mark]

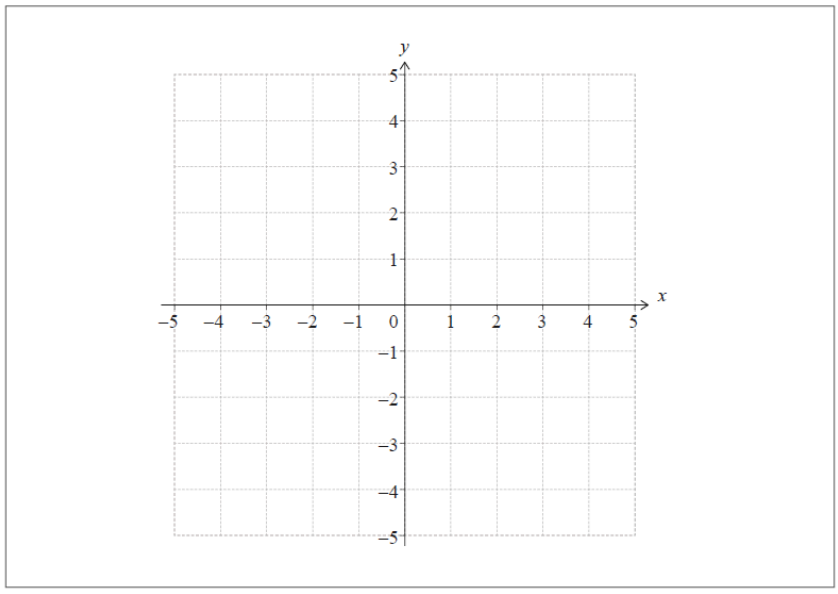
(b.ii) the x-axis. [1]

Markscheme

$(3, 0)$ ***A1***

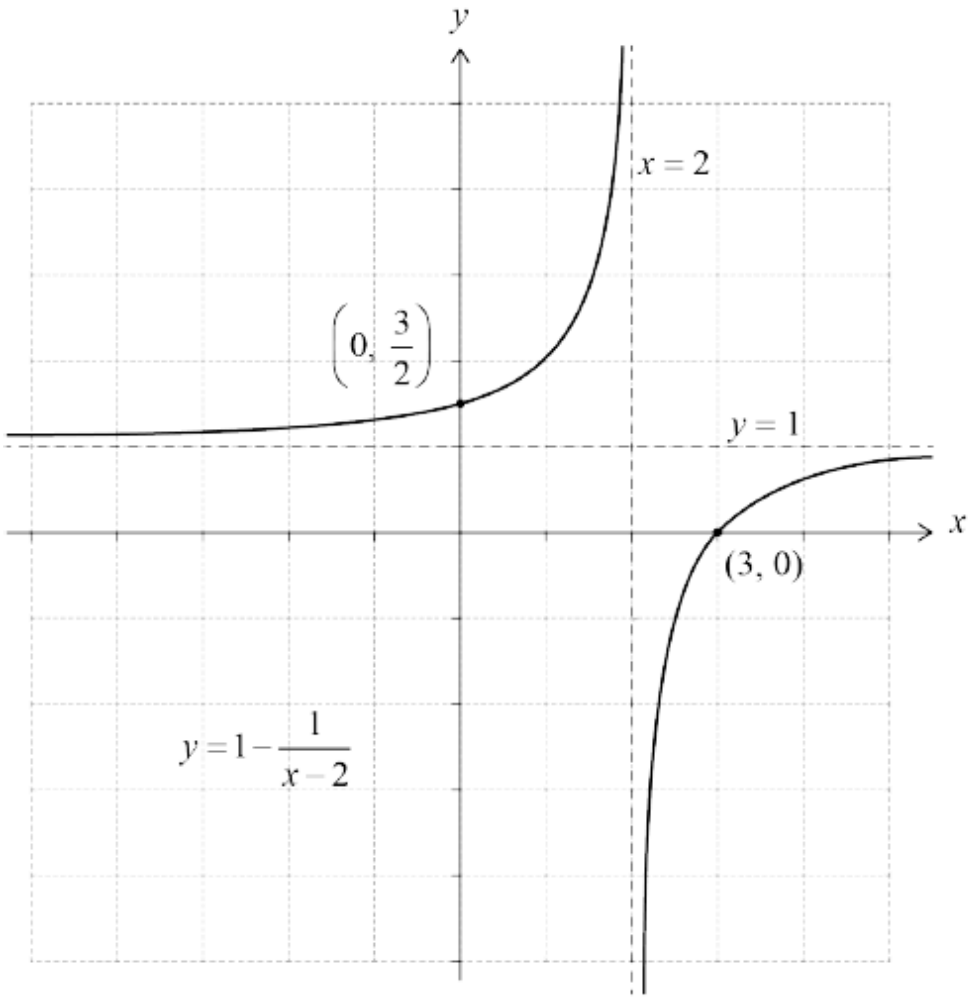
[1 mark]

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).



[1]

Markscheme



two correct branches with correct asymptotic behaviour and intercepts clearly shown **A1**

[1 mark]

17. [Maximum mark: 6] 22M.1.SL.TZ2.4

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

Markscheme

$x = -1$ **A1**

[1 mark]

(a.ii) Write down the equation of the horizontal asymptote. [1]

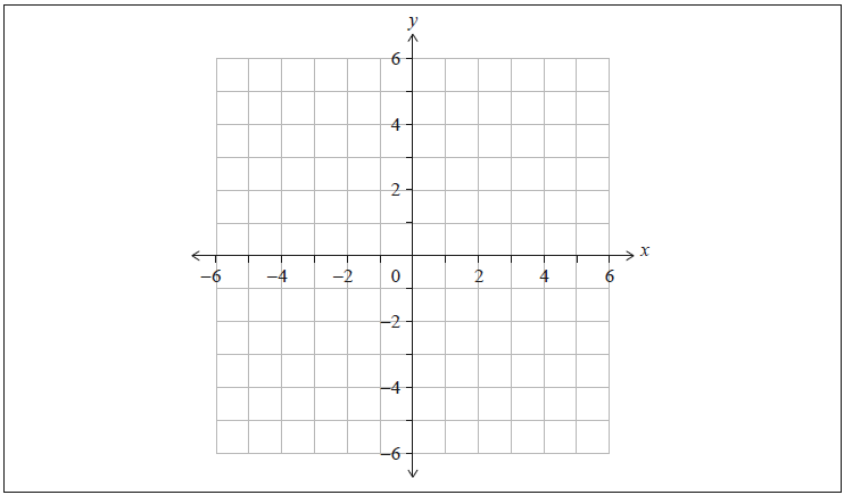
Markscheme

$y = 2$ **A1**

[1 mark]

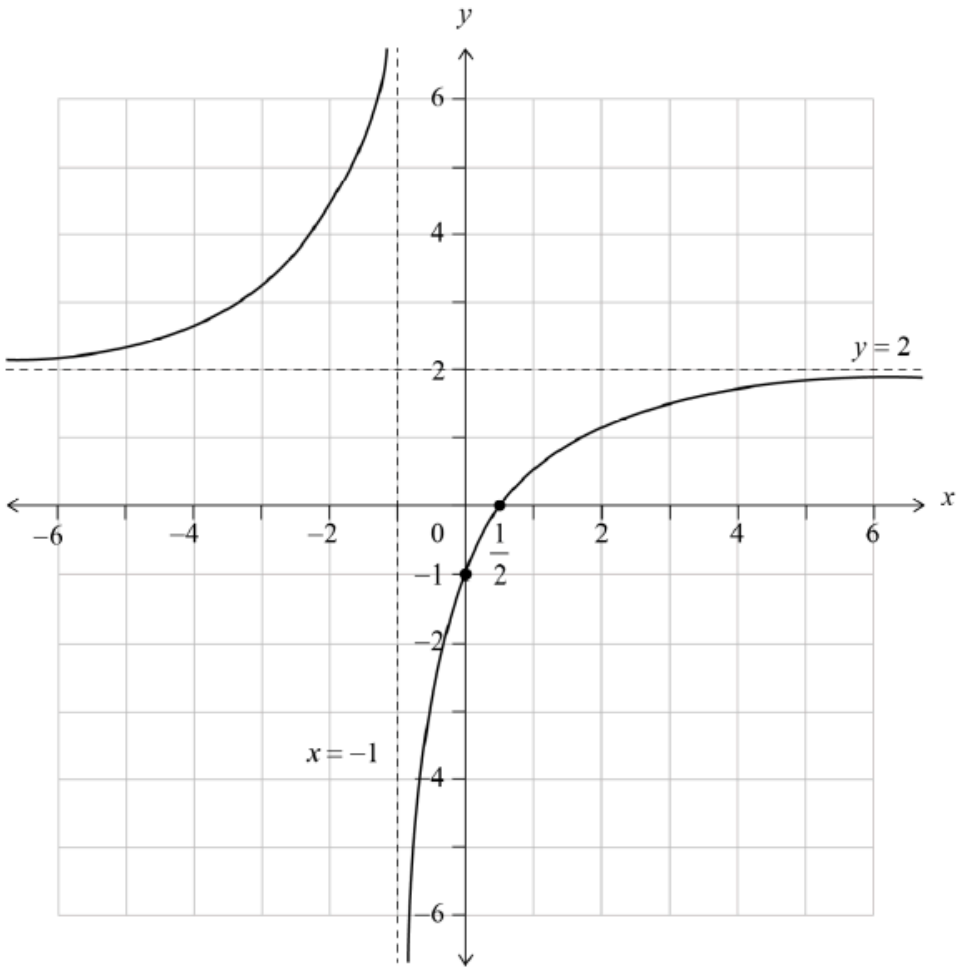
(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

Markscheme



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown

A1

Note: The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x = -1$ and $y = 2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$ **A1A1**

[3 marks]

(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. [1]

Markscheme

$x > \frac{1}{2}$ **A1**

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2},\ \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

18. [Maximum mark: 8] 22M.1.AHL.TZ2.3

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

Markscheme

$x = -1$ **A1**

[1 mark]

(a.ii) Write down the equation of the horizontal asymptote. [1]

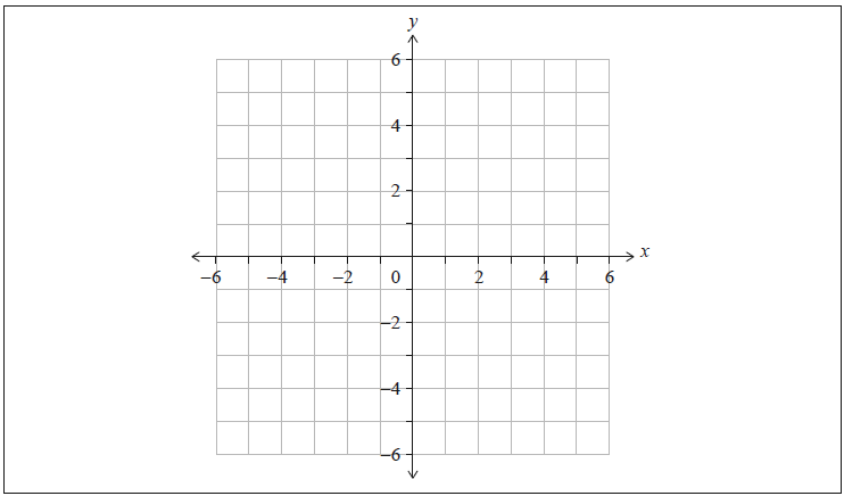
Markscheme

$y = 2$ ***A1***

[1 mark]

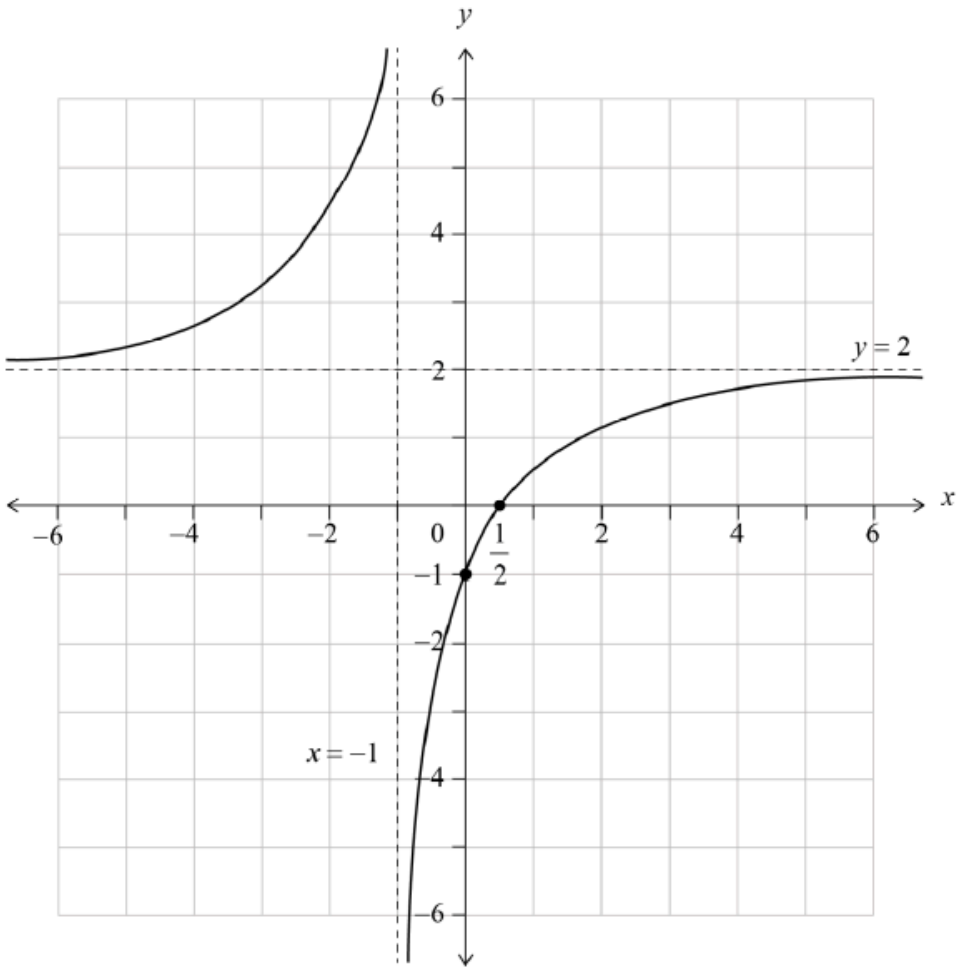
(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

Markscheme



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$ ***A1A1***

[3 marks]

(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. [1]

Markscheme

$x > \frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2},\ \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

(d) Solve the inequality $0 < \frac{2|x|-1}{|x|+1} < 2$. [2]

Markscheme

EITHER

attempts to sketch $y = \frac{2|x|-1}{|x|+1}$ ***(M1)***

OR

attempts to solve $2|x| - 1 = 0$ ***(M1)***

Note: Award the ***(M1)*** if $x = \frac{1}{2}$ and $x = -\frac{1}{2}$ are identified.

THEN

$x < -\frac{1}{2}$ or $x > \frac{1}{2}$ ***A1***

Note: Accept the use of a comma. Condone the use of 'and'. Accept correct alternative notation.

[2 marks]

19. [Maximum mark: 16] 22M.2.SL.TZ1.8

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R},\ x \neq -4$.

For the graph of f

(a.i) write down the equation of the vertical asymptote. [1]

Markscheme

$x = -4$ ***A1***

[1 mark]

(a.ii) find the equation of the horizontal asymptote. [2]

Markscheme

attempt to substitute into $y = \frac{a}{c}$ OR table with large values of x OR sketch of f showing asymptotic behaviour ***(M1)***

$y = 4$ ***A1***

[2 marks]

(b.i) Find $f^{-1}(x)$. [4]

Markscheme

$y = \frac{4x+1}{x+4}$

attempt to interchange x and y (seen anywhere) ***M1***

$xy + 4y = 4x + 1$ OR $xy + 4x = 4y + 1$ ***(A1)***

$xy - 4x = 1 - 4y$ OR $xy - 4y = 1 - 4x$ ***(A1)***

$f^{-1}(x) = \frac{1-4x}{x-4}$ (accept $y = \frac{1-4x}{x-4}$) ***A1***

[4 marks]

(b.ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. [4]

Markscheme

reflection in y-axis given by $f(-x)$ ***(M1)***

$f(-x) = \frac{-4x+1}{-x+4}$ ***(A1)***

reflection of their $f(-x)$ in x-axis given by $-f(-x)$ accept "now $-f(x)$" ***M1***

$(-f(-x) =) \ - \frac{-4x+1}{-x+4}$

$= \frac{-4x+1}{x-4}$ OR $\frac{4x-1}{-x+4}$ ***A1***

$= \frac{1-4x}{x-4} \ \left(= f^{-1}(x)\right)$ ***AG***

Note: If the candidate attempts to show the result using a particular coordinate on the graph of f rather than a general coordinate on the graph

of f, where appropriate, award marks as follows:
***M0A0* for eg** $(2, 3) \to (-2, 3)$
***M0A0* for** $(-2, 3) \to (-2, -3)$

[4 marks]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

(c.i) Find the value of p and the value of q. [2]

Markscheme

attempt to solve $f(x) = f^{-1}(x)$ using graph or algebraically ***(M1)***

$p = -1$ AND $q = 1$ ***A1***

Note: Award ***(M1)A0*** if only one correct value seen.

[2 marks]

(c.ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1}. [3]

Markscheme

attempt to set up an integral to find area between f and f^{-1} ***(M1)***

$\int_{-1}^{1} \left(\frac{4x+1}{x+4} - \frac{1-4x}{x-4} \right) \mathrm{d}\, x$ ***(A1)***

$= 0.675231\ldots$

$= 0.675$ **A1**

[3 marks]

20. [Maximum mark: 5] 21N.1.SL.TZ0.3

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$.

Write down the equation of

(a.i) the vertical asymptote of the graph of f. [1]

Markscheme

$x = 3$ **A1**

[1 mark]

(a.ii) the horizontal asymptote of the graph of f. [1]

Markscheme

$y = -2$ **A1**

[1 mark]

Find the coordinates where the graph of f crosses

(b.i) the x-axis. [1]

Markscheme

$(-2, 0)$ (accept $x = -2$) **A1**

[1 mark]

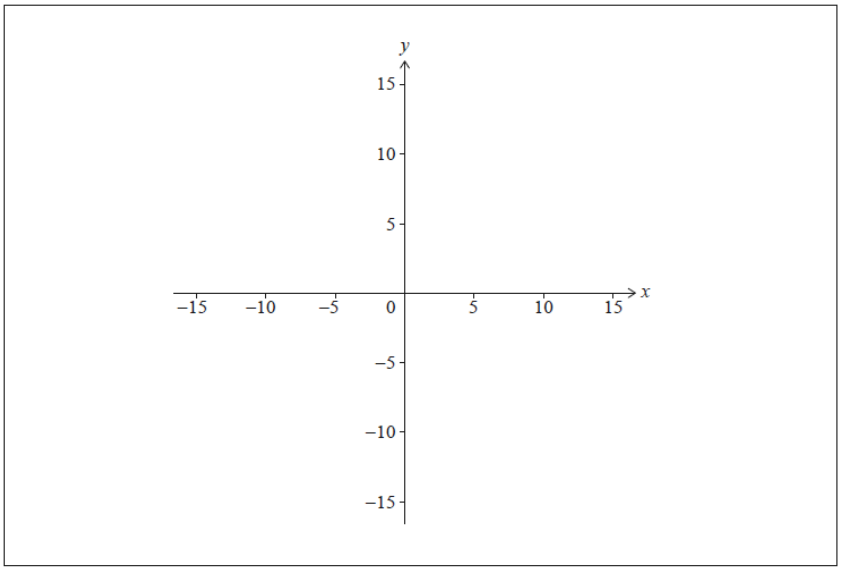
(b.ii) the y-axis. [1]

Markscheme

$\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$) **A1**

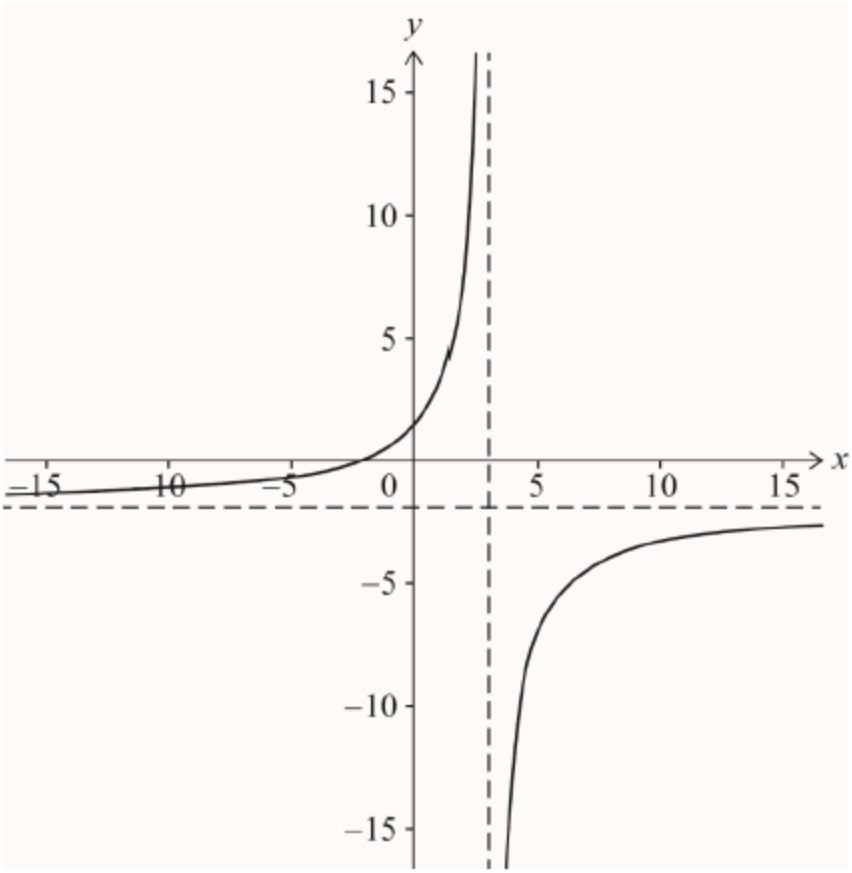
[1 mark]

(c) Sketch the graph of f on the axes below.



[1]

Markscheme



A1

Note: Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

21. [Maximum mark: 9] 21N.1.AHL.TZ0.2

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$.

Write down the equation of

(a.i) the vertical asymptote of the graph of f. [1]

Markscheme

$x = 3$ **A1**

[1 mark]

(a.ii) the horizontal asymptote of the graph of f. [1]

Markscheme

$y = -2$ **A1**

[1 mark]

Find the coordinates where the graph of f crosses

(b.i) the x-axis. [1]

Markscheme

$(-2, 0)$ (accept $x = -2$) **A1**

[1 mark]

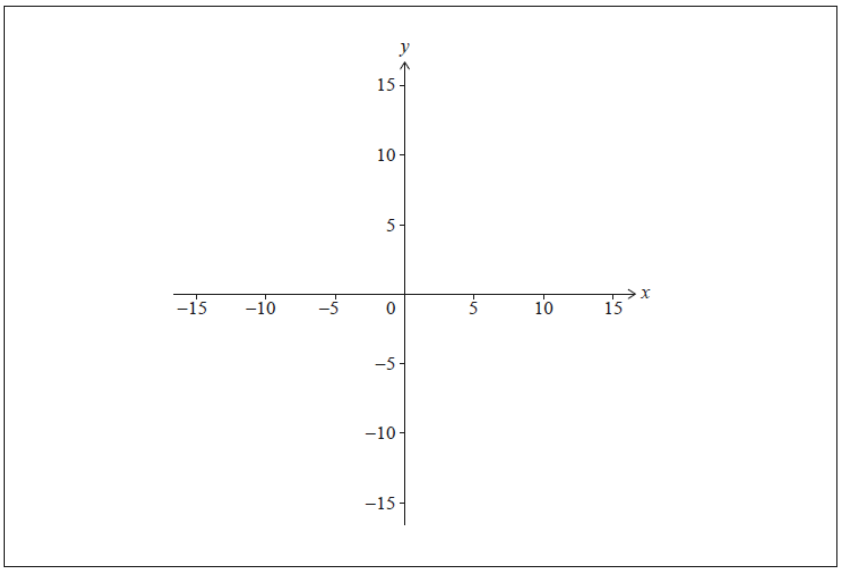
(b.ii) the y-axis. [1]

Markscheme

$\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$) ***A1***

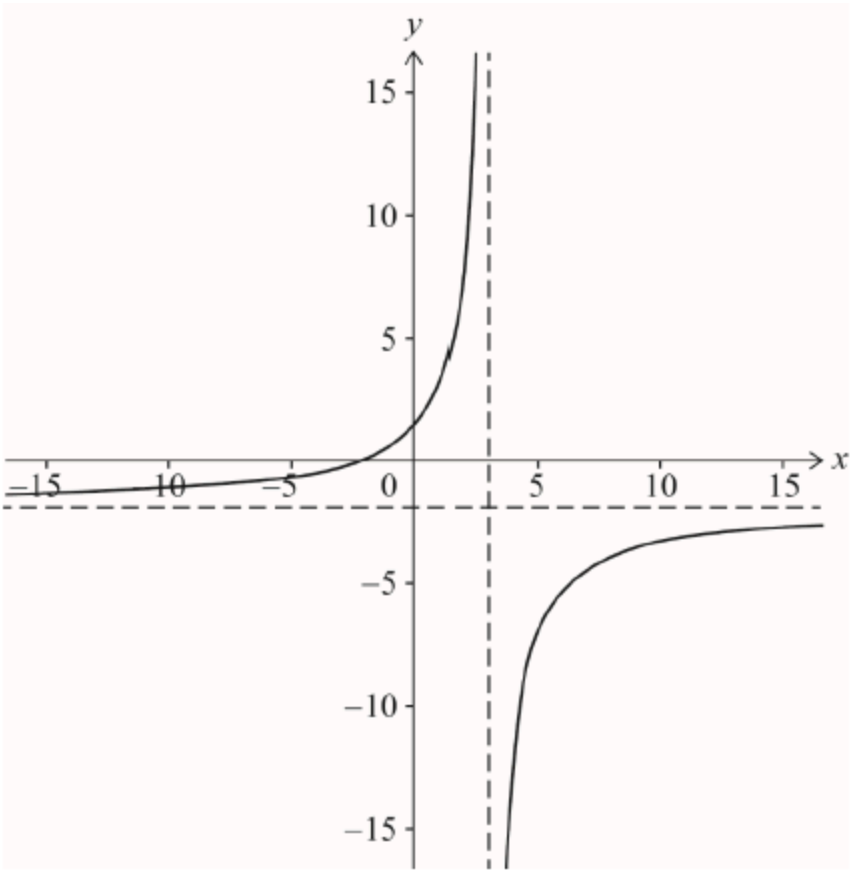
[1 mark]

(c) Sketch the graph of f on the axes below.



[1]

Markscheme



A1

Note: Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

(d) The function g is defined by $g(x) = \frac{ax+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$ and $a \in \mathbb{R}$.

Given that $g(x) = g^{-1}(x)$, determine the value of a. [4]

Markscheme

METHOD 1

$(g(x) =)y = \frac{ax+4}{3-x}$

attempt to find x in terms of y ***(M1)***

OR exchange x and y and attempt to find y in terms of x

$3y - xy = ax + 4$ ***A1***

$ax + xy = 3y - 4$

$x(a + y) = 3y - 4$

$x = \frac{3y-4}{y+a}$

$g^{-1}(x) = \frac{3x-4}{x+a}$ ***A1***

Note: Condone use of $y =$

$g(x) \equiv g^{-1}(x)$

$\frac{ax+4}{3-x} \equiv \frac{3x-4}{x+a}$

$\Rightarrow a = -3$ ***A1***

METHOD 2

$g(x) = \frac{ax+4}{3-x}$

attempt to find an expression for $g(g(x))$ and equate to x ***(M1)***

$gg(x) = \frac{a\left(\frac{ax+4}{3-x}\right)+4}{3-\left(\frac{ax+4}{3-x}\right)} = x$ ***A1***

$\frac{a(ax+4)+4(3-x)}{(9-3x)-(ax+4)} = x$

$\frac{a(ax+4)+4(3-x)}{5-(3+a)x} = x$

$a(ax+4)+4(3-x) = x(5-(3+a)x)$ ***A1***

equating coefficients of x^2 (or similar)

$a = -3$ ***A1***

[4 marks]

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